CALCULATION OF IDEAL FLOW AROUND A SYMMETRIC AEROFOIL USING DIRECT BOUNDARY INTEGRAL EQUATION METHOD WITH LINEAR VARIATION

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ABSTRACT: *In this research paper, a direct boundary integral equation method is used to calculate the ideal flow (i.e. velocity distribution) around a symmetric Aerofoil using linear variation for which the exact result is available. To establish the validity of the method, the outputs given by this computational method are compared with the exact result for ideal flow around the body under consideration.*

Keywords: - Ideal Flow, Symmetric Aerofoil, Direct Boundary Integral Equation Method

INTRODUCTION

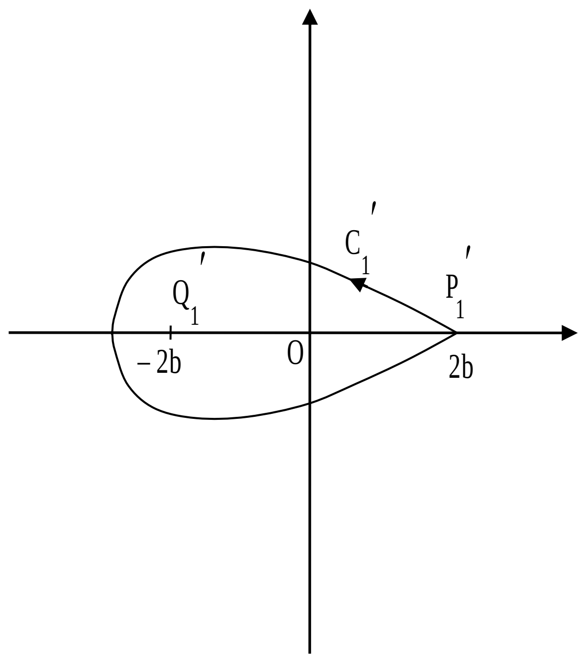
The importance of boundary integral equation method (BIEM) for fluid flow problems have been widely recognized during the last few decades. It is derived through the discretization of an integral equation which is mathematically equals to the original partial differential equation. The benefits of the BIEM are that only the boundary (or boundaries) of the domain of the PDE necessitates the discretization to produce a surface or boundary mesh. Boundary element method also reduces the dimension of the problem by one e.g. an equation in three-dimensional region is transformed into one over its surface and the equation having infinite domain is reduced to an equation over the (finite) boundary. This reduction of dimension leads to smaller linear systems, less computer memory requirements, and more efficient computation. In 1960, with the invention of the computer and the development in the first high level programming language, the approximated numerical solution of BVPs became possible. This method has been successfully applied in a number of fields, e.g., elasticity, potential theory, electrostatics, electrodynamics, magneto hydrodynamics and bio fluid mechanics etc. as detailed in Brebbia et al. [1, 2]. Brebbia [1,2] was inventor of the term 'boundary elements'. The BIEM can be classified in to two categories i.e direct and indirect methods. The formulation of the equation of the direct method is based on green's theorem as used by Milne-Thomson [4] and Shah [5]. In the past, the calculations for flow field around bodies using direct and indirect methods have been used (see Morino[3], Hess & Smith[6], Luminita [7 & 8], Ali [13], Muhammad[11], Mushtaq[9,10 & 12]). In recent years, these methods have been used in calculating the flow field calculations. Thus there is a necessity to apply the DBIEM to calculate the ideal flow field around arbitrary body i.e. symmetric aerofoil and compare the results with analytic solution.

Ideal flow over a Symmetric aerofoil

The modulus of the analytical velocity distribution around a Sy. Aerofoil (see Mushtaq [12].) is given by

$$V = U \left| \frac{1-\left(\frac{r}{z_1-c}\right)^2}{1-\left(\frac{b}{z_1}\right)^2} \right| \quad (1)$$

Where r = circular cylinder (C.C.) radius
a = constant of joukowski transformation
And b = a – r = abscissa of the centre of the C.C.



In Cartesian coordinates, equation becomes

$$V = U\,\frac{\sqrt{[\{(x_1-c)^2+y_1^2\}^2-r^2\{(x_1-c)^2-y_1^2\}]^2+4r^4\,y_1^2(x_1-c)^2}}{[(x_1-c)^2+y_1^2]^2} \times \frac{\sqrt{[(x_1^2+y_1^2)^2-b^2(x_1^2-y_1^2)]^2+4b^2\,x_1^2y_1^2}}{(x_1^2+y_1^2)^2+b^4-2b^2(x_1^2-y_1^2)}$$

Process of discretization

Now consider the case where the symmetric aerofoil's boundary is discretized into linear elements. In this case the nodes are at the intersection of the elements where the boundary conditions are precised.

Divide the surface of the C.C. in the direction of clockwise into n elements by applying the formula (Muhammad [11], Mushtaq [12]).

$$\theta_m = [(n+2) - 2m]\frac{\pi}{n} \qquad m = 1, 2, 3, \ldots n \qquad (2)$$

∴ From equation (2) the end points of these n elements of C.C. can be found. i.e.

$$\gamma_m = -c + r\cos\theta_m$$
$$\delta_m = r\sin\theta_m$$

Now using Joukowski transformation, the extreme points of symmetric aerofoil are

$$z_{1m} = \alpha_m + \frac{b^2}{\alpha_m} \quad (3)$$

Now if $\alpha_m = \gamma_m + i\delta_m$ and $z_{1m} = x_{1m} + iy_{1m}$ then (3) becomes

$$x_{1m} + iy_{1m} = \gamma_m + i\delta_m + \frac{b^2}{\gamma_m + i\delta_m}$$

$$x_{1m} + iy_{1m} = \gamma_m + i\delta_m + \frac{b^2}{\gamma_m + i\delta_m} \times \frac{\gamma_m - i\delta_m}{\gamma_m - i\delta_m}$$

$$x_{1m} + iy_{1m} = \gamma_m + i\delta_m + \frac{b^2(\gamma_m - i\delta_m)}{(\gamma_m + i\delta_m)(\gamma_m - i\delta_m)}$$

$$x_{1m} + iy_{1m} = \gamma_m + i\delta_m + \frac{b^2(\gamma_m - i\delta_m)}{\gamma_m^2 + \delta_m^2}$$

$$x_{1m} + iy_{1m} = \left(\gamma_m + \frac{b^2\gamma_m}{\gamma_m^2 + \delta_m^2}\right) + i\left(\delta_m - \frac{b^2\delta_m}{\gamma_m^2 + \delta_m^2}\right)$$

$$x_{1m} + iy_{1m} = \left(1 + \frac{b^2}{\gamma_m^2 + \delta_m^2}\right)\gamma_m + i\left(1 - \frac{b^2}{\gamma_m^2 + \delta_m^2}\right)\delta_m$$

Comparing real and imaginary parts, we have

$$\left.\begin{aligned} x_{1m} &= \left(1 + \frac{b^2}{\gamma_m^2 + \delta_m^2}\right)\gamma_m \\ y_{1m} &= \left(1 - \frac{b^2}{\gamma_m^2 + \delta_m^2}\right)\delta_m \end{aligned}\right\} \quad m = 1,2,3,\ldots n$$

Boundary condition

In this case the boundary condition over the surface of symmetric aerofoil as follows

$$\frac{\partial \phi_{S.a}}{\partial n} = U \frac{(x_1 + c)}{\sqrt{(x_1 + c)^2 + {y_1}^2}} = \frac{(x_1 + c)}{\sqrt{(x_1 + c)^2 + {y_1}^2}} \quad \text{taking } U = 1 \quad (4)$$

For this case, the equation of DBIEM can be written as

$$-d_i\,\phi_i + \sum_{j=1}^{n} \int_{\sigma_{j-i}} \phi \frac{\partial}{\partial n}\left(\frac{1}{2\pi}\log\frac{1}{r}\right) d\sigma + \phi_\infty = \sum_{j=1}^{n} \int_{\sigma_j} \frac{\partial \phi}{\partial n}\left(\frac{1}{2\pi}\log\frac{1}{r}\right) d\sigma \quad (5)$$

Since ϕ and $\frac{\partial \phi}{\partial n}$ differ linearly over the element, their values are defined at each point on the element in the forms of nodal values and the shape functions M_1 and M_2 as

$$\phi = [M_1\ M_2] \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}$$

$$\frac{\partial \phi}{\partial n} = [M_1\ M_2] \begin{Bmatrix} \frac{\partial \phi_1}{\partial n} \\ \frac{\partial \phi_2}{\partial n} \end{Bmatrix} \quad (6)$$

Now integrals along an element 'j' on L.H.S. of equation (5) can be modified as

$$\int_{\sigma_{j-i}} \phi \frac{\partial}{\partial n}\left(\frac{1}{2\pi}\log\frac{1}{r}\right) d\sigma = \int_{\sigma_{j-i}} [M_1\ M_2] \frac{\partial}{\partial n}\left(\frac{1}{2\pi}\log\frac{1}{r}\right) d\sigma \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}$$

$$= [\,k_{ij}^1\ \ k_{ij}^2\,] \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}$$

Where $k_{ij}^m = \int_{\sigma_{j-i}} M_m \frac{\partial}{\partial n}\left(\frac{1}{2\pi}\log\frac{1}{r}\right) d\sigma$, m = 1,2 (7)

The integrals on R.H.S. of equation (5) can be written as

$$\int_{\sigma_j} \frac{\partial \phi}{\partial n}\left(\frac{1}{2\pi}\log\frac{1}{r}\right) d\sigma = \int_{\sigma_j} [M_1\ M_2]\left(\frac{1}{2\pi}\log\frac{1}{r}\right) d\sigma \begin{Bmatrix} \frac{\partial \phi_1}{\partial n} \\ \frac{\partial \phi_2}{\partial n} \end{Bmatrix}$$

$$= [\,l_{ij}^1\ \ l_{ij}^2\,] \begin{Bmatrix} \frac{\partial\ \phi_1}{\partial n} \\ \frac{\partial\ \phi_2}{\partial n} \end{Bmatrix}$$

Where $l_{ij}^m = \int_{\sigma_j} M_m \left(\frac{1}{2\pi}\log\frac{1}{r}\right) d\sigma$, m = 1,2 n (8)

Beside the panel (i.e element) on the fixed point 'i' lying, the integrals in equations (7) and (8) are calculated numerically as before. These integrals are calculated analytically for this element. Since r and $\hat{n}$ are orthogonal to each other over the element therefore the integrals k_{ij}^1 and k_{ij}^2 are zero. The integrals l_{ij}^1 and l_{ij}^2 have already been calculated (Mushtaq[12]).

The computed velocity distribution is compared with exact solutions for the symmetric aerofoil using FORTRAN programming. The following tables (1-3) and graphs (1-4) show the comparison of the computed velocity with exact velocity over the surface of a symmetric aerofoil for 8, 16, 32, and 64 elements using linear variation

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Table (1)

ELEMENT	X1M	Y1M	R	COMP.VELOCITY	EXACT VELOCITY
1	-2.12	0.42	2.16	0.75262E+00	0.85506E+00
2	-1.52	1.02	1.83	0.18204E+01	0.20233E+01
3	-0.67	1.01	1.22	0.18326E+01	0.20071E+01
4	-0.01	0.42	0.42	0.83342E+00	0.69334E+00
5	-0.01	-0.42	0.42	0.83342E+00	0.69334E+00
6	-0.67	-1.01	1.22	0.18326E+01	0.20071E+01
7	-1.52	-1.02	1.83	0.18204E+01	0.20233E+01
8	-2.12	-0.42	2.16	0.75262E+00	0.85506E+00

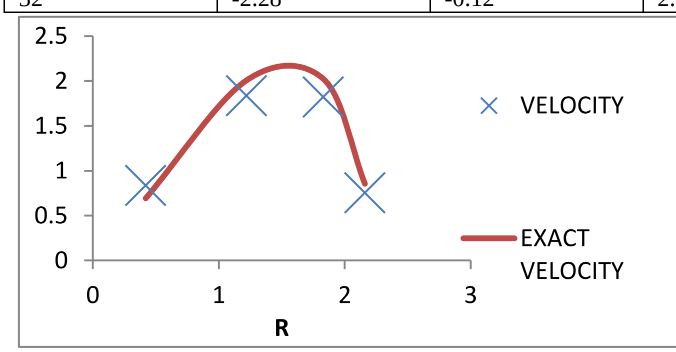
Table (2)

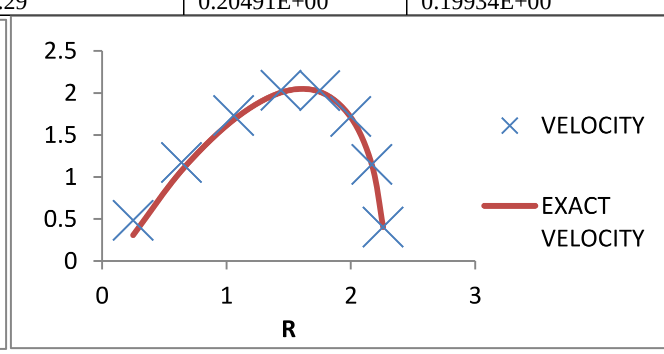
ELEMENT	X1M	Y1M	R	COMP.VELOCITY	EXACT VELOCITY
1	-2.25	0.23	2.26	0.40282E+00	0.40389E+00
2	-2.07	0.65	2.17	0.11473E+01	0.11428E+01
3	-1.75	0.98	2	0.17179E+01	0.17086E+01
4	-1.32	1.15	1.75	0.20281E+01	0.20124E+01
5	-0.87	1.15	1.44	0.20317E+01	0.20049E+01
6	-0.44	0.97	1.06	0.17298E+01	0.1682E+01
7	-0.11	0.63	0.64	0.1172E+01	0.10768E+01
8	0.12	0.22	0.25	0.4856E+00	0.30788E+00

9	0.12	-0.22	0.25	0.4856E+00	0.30788E+00
10	-0.11	-0.63	0.64	0.1172E+01	0.10768E+01
11	-0.44	-0.97	1.06	0.17298E+01	0.1682E+01
12	-0.87	-1.15	1.44	0.20317E+01	0.20049E+01
13	-1.32	-1.15	1.75	0.20281E+01	0.20124E+01
14	-1.75	-0.98	2	0.17179E+01	0.17086E+01
15	-2.07	-0.65	2.17	0.11473E+01	0.11428E+01
16	-2.25	-0.23	2.26	0.40282E+00	0.40389E+00

Table (3)

ELEMENT	X1M	Y1M	R	COMP.VELOCITY	EXACT VELOCITY
1	-2.28	0.12	2.29	0.20491E+00	0.19934E+00
2	-2.24	0.35	2.26	0.60688E+00	0.58825E+00
3	-2.15	0.56	2.22	0.98558E+00	0.95492E+00
4	-2.02	0.76	2.16	0.13265E+01	0.12848E+01
5	-1.85	0.92	2.07	0.16166E+01	0.15651E+01
6	-1.66	1.05	1.96	0.18448E+01	0.17848E+01
7	-1.44	1.14	1.84	0.20024E+01	0.19352E+01
8	-1.21	1.18	1.69	0.20833E+01	0.20102E+01
9	-0.98	1.18	1.53	0.20847E+01	0.20066E+01
10	-0.75	1.14	1.36	0.20065E+01	0.19238E+01
11	-0.53	1.04	1.17	0.18521E+01	0.17641E+01
12	-0.34	0.91	0.97	0.16279E+01	0.15324E+01
13	-0.17	0.74	0.76	0.13432E+01	0.12349E+01
14	-0.04	0.54	0.54	0.10112E+01	0.87753E+00
15	0.06	0.31	0.32	0.66308E+00	0.46369E+00
16	0.16	0.09	0.19	0.25341E+00	0.19671E+00
17	0.16	-0.09	0.19	0.25341E+00	0.19671E+00
18	0.06	-0.31	0.32	0.66308E+00	0.46369E+00
19	-0.04	-0.54	0.54	0.10112E+01	0.87753E+00
20	-0.17	-0.74	0.76	0.13432E+01	0.12349E+01
21	-0.34	-0.91	0.97	0.16279E+01	0.15324E+01
22	-0.53	-1.04	1.17	0.18521E+01	0.17641E+01
23	-0.75	-1.14	1.36	0.20065E+01	0.19238E+01
24	-0.98	-1.18	1.53	0.20847E+01	0.20066E+01
25	-1.21	-1.18	1.69	0.20833E+01	0.20102E+01
26	-1.44	-1.14	1.84	0.20024E+01	0.19352E+01
27	-1.66	-1.05	1.96	0.18448E+01	0.17848E+01
28	-1.85	-0.92	2.07	0.16166E+01	0.15651E+01
29	-2.02	-0.76	2.16	0.13265E+01	0.12848E+01
30	-2.15	-0.56	2.22	0.98558E+00	0.95492E+00
31	-2.24	-0.35	2.26	0.60688E+00	0.58825E+00
32	-2.28	-0.12	2.29	0.20491E+00	0.19934E+00





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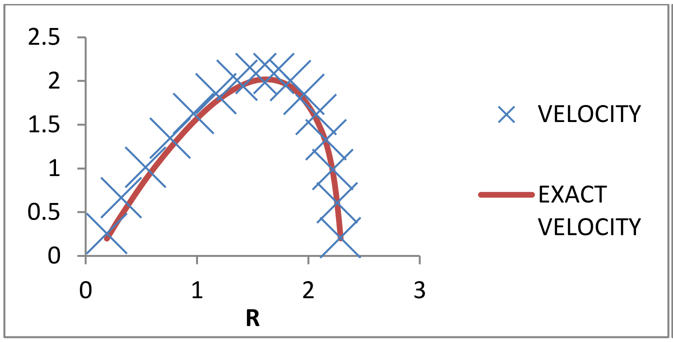

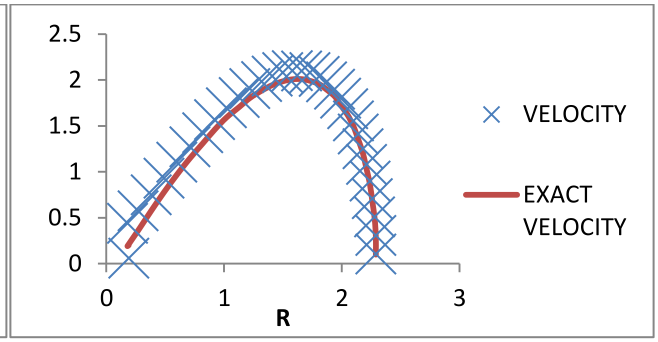

CONCLUSION

A direct boundary integral equation method has been used for the computation of velocity distribution of an ideal flow along two-dimensional body. The computed flow velocity obtained by applying this method is

Compared with exact solutions for flows around the surface of symmetric aerofoil. It is establish that from tables (1-3) and graphs (1-4), the results obtained from the DBIEM for the flow field calculated are in very good agreement with the exact results for the body under consideration.

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